Quaternion Decomposition Based Discriminant Analysis for Color Face Recognition

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***Abstract*—In this paper, we propose a novel *quaternion decomposition based discriminant analysis (QDDA)* method for color face recognition. Unlike traditional approaches that handle color face images by vector representation or by each color channel individually, QDDA makes use of the quaternion to encode all color channels such that we can process all these channels in a holistic way and consider their relations simultaneously. In order to extract more discriminant color information from the image, a decomposition operation is performed to the quaternion matrix. A linear discriminant analysis is finally implemented to the obtained subcomponents for feature extraction. Experimental results have demonstrated the effectiveness of QDDA by comparing with other quaternion based methods.**

I. Introduction

Over the past two decades, color face recognition has attracted a great deal of research interest in the fields of computer vision and pattern recognition. A large number of approaches have been proposed in the literature. These methods usually deal with the RGB color face image in a lower-dimensional space [1], [2] or other color spaces [3]–[5] for better classification results. Although encouraging results have been achieved, these methods are usually subject to the following limitation [6]: they are not able to simultaneously handle all channels of the color image. That is, these methods perform classical grayscale image processing algorithms to each channel individually. This limitation may restrict their performances for color face recognition.

Recently, increasing attention has been paid on color image processing using the quaternion. Mathematically, the quaternion is a generalization of the 2D dimension complex number system to a 4D dimension number system [7]. Therefore, we can encode all channels of a color pixel by one quaternion, which is referred to as the *quaternion representation (QR)* of a color image [8]. In this way, we are able to handle all channels of color images in a holistic way, and consider their relations simultaneously. QR has been successfully applied to many color image processing applications, like object recognition [9], [10], person reidentification [11], synthetic-aperture radar (SAR) image analysis [12], local feature extraction [13], [14], image quality evaluation [15], and segmentation [16].

In order to address the aforementioned limitation of color face recognition, some researchers developed color face recognition algorithms making use of QR from different perspectives [17]–[23]. Among these methods, several classical discriminant analysis methods, including principal component analysis (PCA), linear discriminant analysis (LDA) and 2D PCA, have been extended into quaternionic domain [6], [19], [20]. Similarly, Wu proposed a quaternion-based improved locality preserving projection (LPP) for color face recognition [23]. These methods are finally transformed to find the eigenvectors and eigenvalues of a quaternion matrix, and promising results have been achieved.

However, most of existing quaternion based discriminant analysis schemes directly work on the quaternion matrix. The matrix consists of all color information of the original image, but it also contains several negative information, like the noise or illumination variations. The features, derived from the unwanted information, may adversely affect the performance of a specific application. It is necessary to remove the useless information from the quaternion matrix to gain more robust characteristics for feature extraction.

In this paper, we propose a novel discriminant analysis method for color face recognition using QR. Unlike existing methods, which directly deal with the quaternion matrix, the proposed method first decomposes the quaternion matrix into two components. These components are parallel and vertical to a previously given quaternion [24]. Therefore, the proposed method is referred to as *quaternion decomposition based discriminant analysis (QDDA)*. By decomposing the QR of color images by a specific direction in the color space, different characteristics of the color information can be extracted which may convey more discriminant features for face recognition. After quaternion decomposition, a linear discriminant analysis is performed to the obtained subcomponents for feature extraction. Comparison results demonstrate that the proposed QDDA outperforms other quaternion based discriminant analysis methods in most test cases.

The remainder of the paper is organized as follows. In Section 2 we introduce the related mathematical background about quaternion algebra. In Section 3 we describe the proposed QDDA algorithm in detail. In Section 4 several experiments are carried out to evaluate QDDA. Section 5 offers our conclusions.

II. Preliminaries

A. Quaternion Algebra

The quaternion, created by Hamilton in 1843 [7], is a four-dimensional generalization of the complex number system

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with one real part and three imaginary parts. A quaternion $\dot{q} \in \mathbb{H}$ can be represented in a complex form as follows:

$$\dot{q} = q + ia + jb + kc, \tag{1}$$

where q, a, b and c are real numbers, and $\{i, j, k\}$ are complex operators satisfying:

$$i^2 = j^2 = k^2 = ijk = -1, \tag{2}$$

$$ij = -ji = k, \tag{3}$$

$$jk = -kj = i, \tag{4}$$

$$ki = -ik = j. \tag{5}$$

q is the real part of $\dot{q}$, and $ia + jb + kc$ is the imaginary part. If $q = 0$, $\dot{q}$ is a pure quaternion.

Several basic properties of quaternion can be similarly derived as the complex number system. For instance, the conjugate and modulus of $\dot{q}$ are defined as follows:

$$\dot{q}^* = q - (ia + jb + kc), \tag{6}$$

$$|\dot{q}| = \sqrt{\dot{q}\dot{q}^*} = \sqrt{\dot{q}^*\dot{q}} = \sqrt{q^2 + a^2 + b^2 + c^2}. \tag{7}$$

Note that $\dot{q}$ is a unit quaternion if $|\dot{q}| = 1$. More details about quaternion algebra can be found in [7], [8].

B. Eigenvalues and Eigenvectors for Quaternion Matrices

Similar to the real matrix, the eigenequation of a quaternion matrix $\dot{S}$ is defined as:

$$\dot{S}\dot{\boldsymbol{u}} = \dot{\lambda}\dot{\boldsymbol{u}}, \tag{8}$$

where $\dot{\boldsymbol{u}}$ and $\dot{\lambda}$ is the eigenvector and eigenvalue of $\dot{S}$. Many quaternion based discriminant analysis methods finally turn to find the eigenvectors corresponding to the largest eigenvalues. In order to address Eq. (8), $\dot{S}$ is first transformed into its equivalent complex matrix, then we solve the eigenvalues and eigenvectors of the complex matrix instead. For more details, see [25].

III. Methodology

The proposed approach mainly consists of three steps: QR of color images, quaternion decomposition, and quaternion based discriminant analysis. In the following, we will detail these steps respectively.

A. QR of Color Images

To handle the color image in quaternionic domain, the first step is to represent it by quaternions. However, the color image is usually described in RGB color space, which is a 3D space, while the quaternion is a 4D number system. To remove this mismatch between color space and the quaternionic domain, the imaginary part of a quaternion is used to represent a color pixel as follows [8]:

$$\dot{Q}(x, y) = iR(x, y) + jG(x, y) + kB(x, y), \tag{9}$$

where $\dot{Q}(x, y)$ is QR of the color pixel, and $R(x, y)$, $G(x, y)$, and $B(x, y)$ are the red, green, and blue components of a

TABLE I

Decomposition results of $\dot{q}$ with different directions.

$\dot{p}$	$\dot{q}_{\shortparallel}$	$\dot{q}_{\perp}$
$\dot{p}_1$	ri	$gj + bk$
$\dot{p}_2$	$\frac{r+g}{2}i + \frac{r+g}{2}j$	$\frac{r-g}{2}i + \frac{g-r}{2}j + bk$
$\dot{p}_3$	$\frac{r+g+b}{3}i + \frac{r+g+b}{3}j + \frac{r+g+b}{3}k$	$\frac{2r-g-b}{3}i + \frac{2g-r-b}{3}j + \frac{2b-r-g}{3}k$
$\dot{p}_4$	$\frac{r+2g+b}{6}i + \frac{r+2g+b}{3}j + \frac{r+2g+b}{6}k$	$\frac{5r-2g-b}{6}i + \frac{g-b-r}{3}j + \frac{5b-r-2g}{6}k$

(a) (b) (c)

Fig. 1. The parallel components of decomposition results for color Lena image using different decomposition directions given in Table I respectively: (a) $\dot{p}_2 = \frac{i+j}{\sqrt{2}}$, (b) $\dot{p}_3 = \frac{i+j+k}{\sqrt{3}}$, and (c) $\dot{p}_4 = \frac{i+2j+k}{\sqrt{6}}$.

color pixel respectively. $\dot{Q}(x, y)$ gives a one-to-one mapping between the quaternionic domain and RGB color space. Any operations to $\dot{Q}(x, y)$ will affect all the color channels at the same time.

B. Quaternion Decomposition

Ell and Sangwine [24] pointed out that a pure quaternion $\dot{q}$ can be decomposed into two components which are parallel and vertical to a given quaternion $\dot{p}$. Denoting these two components of $\dot{q}$ by $\dot{q}_{\shortparallel}$ and $\dot{q}_{\perp}$ respectively, we have: $\dot{q}_{\shortparallel} \parallel \dot{p}$, $\dot{q}_{\perp} \perp \dot{p}$, and $\dot{q} = \dot{q}_{\shortparallel} + \dot{q}_{\perp}$. They can be obtained as follows [24]:

$$\dot{q}_{\shortparallel} = 0.5 * (\dot{q} + \dot{p}\dot{q}\dot{p}), \tag{10}$$

$$\dot{q}_{\perp} = 0.5 * (\dot{q} - \dot{p}\dot{q}\dot{p}). \tag{11}$$

In color space, different characteristics of the color pixel can be achieved by choosing a proper decomposition direction. Suppose that $\dot{q} = ri + gj + bk$. We consider four special directions in the color space: $\dot{p}_1 = i$, $\dot{p}_2 = \frac{i+j}{\sqrt{2}}$, $\dot{p}_3 = \frac{i+j+k}{\sqrt{3}}$, and $\dot{p}_4 = \frac{i+2j+k}{\sqrt{6}}$. The corresponding decomposition results of $\dot{q}$ are given in Table I, and the parallel components of color Lena image are illustrated in Fig. 1 as an example. We can observe that the direction determined by $\dot{p}_1$ can portion the red component from all color channels. For the results obtained by $\dot{p}_2$ and $\dot{p}_3$, the average values of corresponding color channels are extracted. To $\dot{p}_4$, $\dot{q}_{\shortparallel}$ is the weighted averages of all color channels of $\dot{q}$. Therefore, $\dot{q}_{\shortparallel}$ can be regarded as a special type of "mean" value of $\dot{q}$, while the remainder part $\dot{q}_{\perp}$ represents the "mean" removal results. They contain different color information of the original color image.

C. QDDA Method

Based on the decomposition method described in Section III-B, the QR of a color image $\dot{Q}$ is decomposed into two parts: $\dot{Q}_{\shortparallel}$ and $\dot{Q}_{\perp}$. Then some existing quaternion-based discriminant analysis approaches, such as QPCA, QLDA, and QLPP, can be performed to the decomposition results for feature extraction. In this work, the QLDA [6] is used in QDDA method as an example.

Suppose that there exist totally T training samples for L classes in the training set. Take QDDA using the vertical component of the QR as an example. For tth training sample, convert $\dot{Q}_{t_\perp}$ into a quaternion vector $\dot{\boldsymbol{q}}_{t_\perp}$ by concatenating the rows in sequence. Then calculate the generative matrix as follows:

$$\dot{M}_\perp = \frac{1}{L}\sum_{l=1}^{L}(\bar{\dot{\boldsymbol{q}}}_{l_\perp} - \bar{\dot{\boldsymbol{q}}}_\perp)(\bar{\dot{\boldsymbol{q}}}_{l_\perp} - \bar{\dot{\boldsymbol{q}}}_\perp)^H, \tag{12}$$

where H means conjugate transpose, $\bar{\dot{\boldsymbol{q}}}_{l_\perp}$ stands for the average value of the training samples of the lth class, and $\bar{\dot{\boldsymbol{q}}}_\perp$ represents the average value of all training samples. Similar to the discriminant analysis methods in real number domain, we find the eigenvalues and eigenvectors of the quaternion matrix $\dot{M}_\perp$, then select the eigenvectors that are corresponding to the first d largest eigenvalues as the transform axes. Concatenate these d eigenvectors together to form a transform matrix $\dot{\Psi}_\perp$. The detail derivation of $\dot{\Psi}_\perp$ is summarized in Algorithm 1, and the approach presented here is referred to as QDDA. The features of $\dot{\boldsymbol{q}}_{t_\perp}$ are then extracted by projecting it onto $\dot{\Psi}_\perp$ as follows:

$$\boldsymbol{f}_{t_\perp} = \dot{\boldsymbol{q}}_{t_\perp}\dot{\Psi}_\perp. \tag{13}$$

We can obtain $\boldsymbol{f}_{t_\parallel}$, the features from $\dot{\boldsymbol{q}}_{t_\parallel}$, in a similar way.

Algorithm 1 The QDDA Algorithm

Input: A set of training color images for L classes, decomposition direction $\dot{q}_0$.

Output: The transform matrix $\dot{\Psi}_\perp$.

- Step 1: Represent the tth color image I_t by a quaternion matrix $\dot{Q}_t$ using Eq. (9);
- Step 2: Based on Eq. (11), extract $\dot{Q}_{t_\perp}$ as the subcomponent of $\dot{Q}_t$ that is vertical to $\dot{q}_0$;
- Step 3: Convert $\dot{Q}_{t_\perp}$ to its vector version $\dot{\boldsymbol{q}}_{t_\perp}$, then calculate $\bar{\dot{\boldsymbol{q}}}_{l_\perp}$ and $\bar{\dot{\boldsymbol{q}}}_\perp$ which are the average values of the lth class and of all training samples;
- Step 4: Compute the generative matrix $\dot{M}_\perp$ as in Eq. (12), then calculate its eigenvectors and eigenvalues;
- Step 5: Concatenate the eigenvectors together which are corresponding to the first d largest eigenvalues to form the transform matrixes $\dot{\Psi}_\perp$.

IV. Experimental results

In this section, experiments will be conducted to evaluate the performances of the proposed QDDA. First, we introduce the experimental setting. Then the effects of quaternion decomposition directions are studied. Finally, QDDA is compared with other quaternion-based discriminant analysis methods.

Fig. 2. Examples of color face images in GTFB databse.

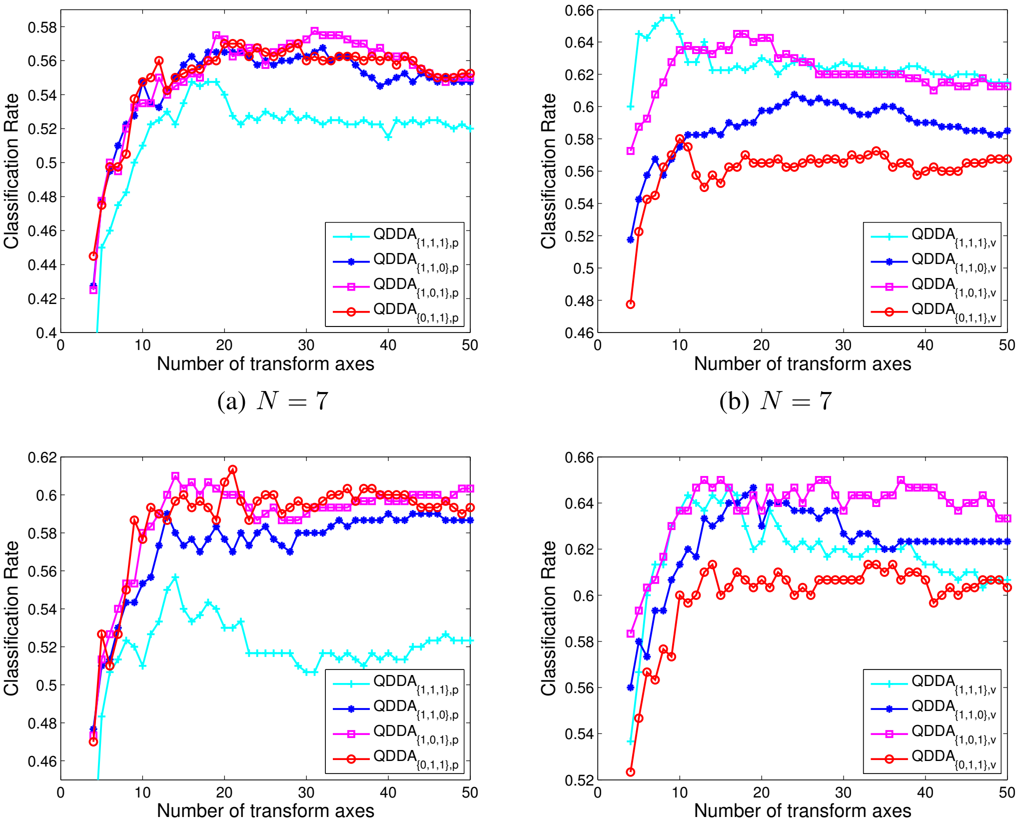
A. Experimental Setting

The normalized Georgia Tech face database (GTFB), created by Xu [6], is selected in our experiments. It includes color face images of 50 people, and there are 15 images for each people. All these face images were manually labeled and extracted from the original images. Each face image is resized to 40×30. Some face images are illustrated in Fig. 2 as examples. Images on the same row are of the same person. We can find that there are variations of facial expressions, lighting conditions, and pose in the images. In all tests, we apply the first N images of each subject to form the training samples and consider the rest images as testing samples, and the number of transform axes is set from 4 to 50. The nearest neighbor (NN) classifier with l_2 norm is chosen here. The correct classification rate is applied to measure the performance of the test methods.

B. The Effects of Quaternion Decomposition Directions

In this experiment, we first study the performances of QDDA using different decomposition directions. As aforementioned, different characteristics of the color pixel can be represented by selecting special decomposition direction. In this work we consider following directions represented by quaternions: $\dot{\mu} = \frac{i+j+k}{\sqrt{3}}$, $\dot{q}_1 = \frac{i+j}{\sqrt{2}}$, $\dot{q}_2 = \frac{i+k}{\sqrt{2}}$, and $\dot{q}_3 = \frac{j+k}{\sqrt{2}}$. $\dot{\mu}$ is the gray line in RGB color space, and $\{\dot{q}_1, \dot{q}_2, \dot{q}_3\}$ are the center lines in corresponding 2D color planes. Denote QDDA on the parallel component of decomposition results using the above directions by: $\text{QDDA}_{\{1,1,1\},p}$, $\text{QDDA}_{\{1,1,0\},p}$, $\text{QDDA}_{\{1,0,1\},p}$, and $\text{QDDA}_{\{0,1,1\},p}$ respectively. The corresponding results of vertical component are represented by $\text{QDDA}_{\{\bullet\},v}$.

The number of training sample N is set to 7 and 9 here. The corresponding results are illustrated in Fig. 3, where (a) and (b) are the results of $\text{QDDA}_{\{\bullet\},p}$ and $\text{QDDA}_{\{\bullet\},v}$ with $N = 7$, and (c) and (d) are corresponding results of $N = 9$. We can find that the performances of $\text{QDDA}_{\{\bullet\},p}$ and $\text{QDDA}_{\{\bullet\},v}$ using the same decomposition direction may quite differ. $\text{QDDA}_{\{1,1,1\},p}$ achieves worst results when $N = 7$, but $\text{QDDA}_{\{1,1,1\},v}$ outperforms $\text{QDDA}_{\{1,1,0\},v}$ and $\text{QDDA}_{\{0,1,1\},v}$. In contrast, $\text{QDDA}_{\{1,1,0\},p}$ achieves satisfying results, but $\text{QDDA}_{\{1,1,0\},v}$ just obtains the third best results in all $\text{QDDA}_{\{\bullet\},v}$. On the whole, the accuracies obtained by vertical component are better than those obtained by parallel components. Among all test decomposition directions, $\dot{q}_2 = \frac{i+k}{\sqrt{2}}$ attains stable and most satisfying performances than other directions.



(a) $N = 7$ (b) $N = 7$

(c) $N = 9$ (d) $N = 9$

Fig. 3. Classification accuracies of QDDA using parallel and vertical components of QR of color images. (a) and (b) are the results of QDDA$_{\{\bullet\},p}$ and QDDA$_{\{\bullet\},v}$ with $N = 7$, and (c) and (d) are corresponding results of $N = 9$.

C. Comparison Results with Other Methods

This experiment evaluates QDDA by comparing with two related methods, namely QPCA and QLDA. The number of training sample N is set to 4, 6, 8, and 10 respectively. The decomposition direction is chosen as $\dot{\mu} = \frac{i+j+k}{\sqrt{3}}$ for QDDA. The corresponding results are given in Fig. 4. We can find that QDDA$_{\{1,1,1\},p}$ performs worse than others methods in most cases. In the situation of $N = 4$, QDDA$_{\{1,1,1\},v}$ and QPCA obtain comparable results, and they keep about 10 percentages higher than QLDA. When N increases to 6, QPCA and QLDA outperform QDDA$_{\{1,1,1\},v}$ in some cases. For larger training numbers ($N = 8, 10$), QLDA shows better performance than QPCA. In these situations, QDDA$_{\{1,1,1\},v}$ obtains obvious improvements comparing with QLDA.

V. Conclusion

This paper proposed QDDA as a novel quaternion based discriminant analysis method for color face recognition. To overcome the limitation of traditional methods, QDDA is based on QR of color images such that all channels of the color images are handled holistically. Besides, a quaternion decomposition operation is carried out in QDDA to extract more discriminant information from the color pixels. Experiments were carried out to evaluate QDDA, and encouraging results have been achieved. Future works include the adaptive selection of decomposition direction for each color image, considering more relations between training samples, and performing other transformations to QR of color image, etc.

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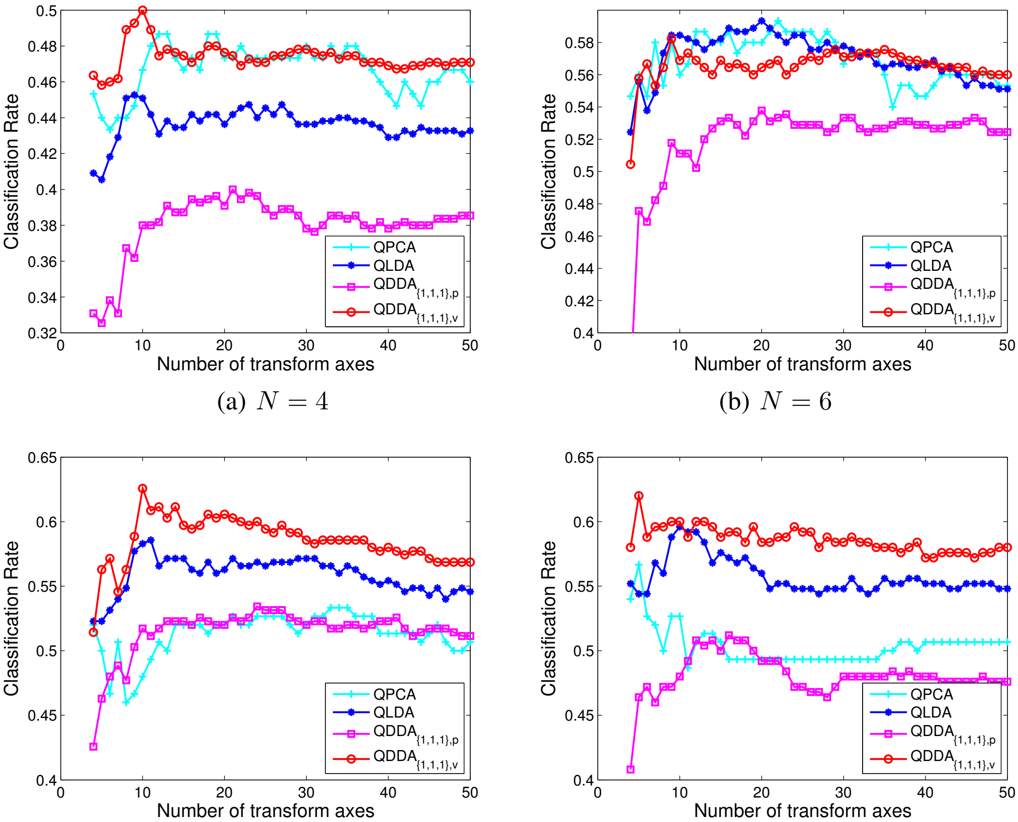

(a) $N = 4$ (b) $N = 6$ (c) $N = 8$ (d) $N = 10$

Fig. 4. Classification accuracies of QDDA, QPCA, and QLDA on the normalized GTFB color face database with different numbers of the training samples.

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